Strong Start Math Project

Tuesday, June 21
8:00 a.m. – 11:30 a.m.
Today’s Agenda

• Welcome!!
• Counting and Cardinality & Operations and Algebraic Thinking Domain
• Ramp Up to Algebra
• Standards for Mathematical Practice
• Early Counting
• Mathematical Curiosity
• MKT Assessment
COMMON CORE STATE STANDARDS FOR MATHEMATICS (CCSSM)

THE THREE KEY SHIFTS
Learning Intentions

We will understand the key shifts, and the structure, of the Common Core State Standards for Mathematics (CCSSM).

Deepen our understanding of the Domain of Operations and Algebraic Thinking.
Background

COMMON CORE STATE STANDARDS FOR Mathematics

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**COMMON CORE STATE STANDARDS FOR Mathematics**

- This page discusses the background information about the Common Core State Standards for Mathematics.

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**Context**

The Common Core State Standards for Mathematics are a set of educational benchmarks for mathematics in the United States. These standards aim to provide a consistent, clear understanding of what students are expected to learn, so educators and parents know what they need to teach in every school. The standards are designed to be challenging yet achievable for all students, promoting deep understanding, problem-solving skills, and the ability to apply mathematical concepts in real-world situations. By focusing on fewer, more rigorous standards, the goal is to ensure that students graduate from high school ready to succeed in college and careers, and fully prepared for the demands of the 21st century.
Released in June 2010... six years ago...

- Who initiated the CCSS?
- Who wrote the CCSS?
- What are the CCSS?
- Why were they written?
Who initiated and led the CCSS?

NGA    National Governors Association
CCSSO   Council of Chief State School Officers

Conversations began in 2007..... in 2009, bipartisan group of governors and state education chiefs in 48 states agreed to lead the development of common standards in ELA and mathematics.


Retrieved from http://www.corestandards.org
Who wrote the CCSS?

NGA/CCSSO established:

- Work teams in mathematics and ELA.
- Feedback groups and review teams ...... including three opportunities for states to review drafts and a public review document.
- All occurring from June 2009 through June 2010.
Three Lead Writers
Common Core State Standards for Mathematics

Bill McCallum  Phil Daro  Jason Zimba

• Charge given to the authors:
• Focused, coherent, clear, and rigorous.
• Anchored in College and Career Readiness.
• Research and evidence-based.
• Internationally benchmarked.

“Fewer, clearer, and higher.”
A Long Overdue Shifting of the Foundation

For as long as most of us can remember, the K-12 mathematics program in the U.S. has been aptly characterized in many rather uncomplimentary ways: underperforming, incoherent, fragmented, poorly aligned, narrow in focus, skill-based, and, of course, “a mile wide and an inch deep.”

---Steve Leinwand, Principal Research Analyst
American Institutes for Research in Washington, D.C
CCSSM KEY SHIFTS
CCSSM Key Shifts: Group Discussion

Count off in your table group, 1-2-3-1-2-3.

1: Focus
2: Coherence
3: Rigor

Read your assigned Shift.

Take turns facilitating a table group discussion:

– Highlight the key ideas for your group.
– Surface questions about the shift.
Focus
Fewer topics, deeply attend to the major work of each grade.

Coherence
Interconnected concepts within a grade, progressions across grades that extend previous learning.

Rigor
Deep, authentic understanding of concepts and procedures (not just harder); can use and apply math knowledge in real-world situations.
CCSSM Structure and Terminology

Standards for Mathematical Practice

• Grades K-12
• 8 Standards of Mathematical Practices

Standards for Mathematical Content

• K–8 Standards by Grade Level
• ___________________________
  – Domains
  • Clusters
    – Standards
### Common Core State Standards

#### K-8 Domains & HS Conceptual Categories

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Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Reasoning and Explaining
Modeling and Using Tools
Seeing Structure and Generalizing

William McCallum, The University of Arizona
COUNTING AND CARDINALITY; OPERATIONS AND ALGEBRAIC THINKING AND THE RAMP UP TO ALGEBRA
## K-8 Domains & HS Conceptual Categories

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### K-8 Domains

- **Operations & Algebraic Thinking**
- **Number & Operations in Base Ten**
- **Measurement & Data**
- **Statistics & Probability**
- **Geometry**

### HS Conceptual Categories

- **Algebra**
- **Number and Quantity**
- **Modeling**

### K-8 Concepts
- Counting & Cardinality

### K-8 Domains & HS Conceptual Categories Overview

This chart outlines the progression of mathematical concepts from K-8 domains to HS conceptual categories, highlighting the key areas of study and their alignment in the educational curriculum.
Counting and Cardinality and Operations and Algebraic Thinking

Read and highlight pages 2-3 of the Progressions document on the Counting and Cardinality and Operations and Algebraic Thinking domains.

! Important Idea

? Idea I don’t understand
Counting and Cardinality; Operations and Algebraic Thinking

What is it?

Word Splash!

Quickly list out any words, ideas, or phrases that come to mind as you think of “Counting and Cardinality” and “Operations and Algebraic Thinking.”
Why an “Operations and Algebraic Thinking” Domain?

• Reflect on your Word Splash in the context of the video.
  – What ideas surfaced in the video that affirmed our word splash?
  – Using a different color marker, let’s add any new ideas to our word splash.

http://youtube.com/watch?v=HMMe8_4s9KE
And in the domain of Operations and Algebraic Thinking, it is those meanings, properties, and uses which are the focus; and it is those meanings, properties, and uses that will remain when students begin doing algebra in middle grades [and beyond].

---Dr. Jason Zimba
The foundation for algebra!

The diagram shows:

- Meanings of the Operations
- Properties of the Operations
- Contextual Situations

These concepts are interconnected as the foundation for algebra.
A Brief Overview of the CC and OA Domain for K-3

- K4 & K5 read Critical Area #1 p. 9 CCSSM
- Grade 1 read Critical Area #1 p. 13 CCSSM
- Grade 2 read Critical Area #2 p. 17 CCSSM
- Grade 3 read Critical Area #1 p. 21 CCSSM

- Be prepared to share with your table group.
K5 Counting and Cardinality; Operations and Algebraic Thinking Progression
Grades K-3

Story Problem Structures and Mathematical Representations

Counting and Cardinality
Number and operation sense
Fluency with addition & subtraction strategies
Fluency with multiplication & division
This progression deals with early counting and “how much is in a group” (cardinality). The progression illustrates the basic operations including the kinds of quantitative relationships they model, and the types of problems that can be solved.

--Council of Great City Schools

http://www.commoncoreworks.org/Page/337
Algebra is considered a “gateway” course, a gate that many students struggle to get through successfully.

What might be some of the reasons so many students hit a wall when they get to algebra?
The Hunt Institute Video Series
Common Core State Standards: A New Foundation for Student Success

http://www.youtube.com/watch?v=ONPADO_Nt14&list=PL913348FFD75155C6&index=22
Ramp Up to Algebra

What are some of the ideas that “ramp up” from the study of number and operations to the study of algebra?
An Initial Frame for Our Thinking

K, Counting and Cardinality; K-5 Operations and Algebraic Thinking

• Focus: Understand and use numbers with meaning
• Students gain experience and develop proficiency in understanding how the operations work and the relationship between the operations. Properties, meanings and uses of the operations take center stage.

K-5 Number and Operations in Base Ten

• Focus: Understand the structure of our number system to compute
• Students gain experience and develop proficiency with computation strategies. In grades K-2 these strategies are based on their understanding of number, place value, and properties of the operations.
Learning Intentions

We will

• Understand the key shifts, and the structure, of the Common Core State Standards for Mathematics (CCSSM).

• Deepen our understanding of the Domain of Operations and Algebraic Thinking.
Please be back in 10 minutes.
STANDARDS FOR MATHEMATICAL PRACTICE
Where are the Standards for Mathematical Practice in the CCSSM document?
Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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William McCallum, The University of Arizona
Content vs. Practices

• Think Math! article: Read the section on “Connecting Mathematical Practice with Content.” Highlight 3 items that catch your attention.

• Round Robin Sharing
  – One person begins by sharing an idea from the reading.
  – Moving to the right, each remaining group member contributes his or her thinking on that same idea. Ideas are offered with no cross talk.
  – After everyone shares, repeat steps 1 and 2 for another idea begun by a new group member.

• Continue this round robin sharing three times.
Jigsaw Reading
Math Practices

• Each table will be assigned one Standard for Mathematical Practice (MP) to read and study.
  – MP1. Make sense of problems and persevere in solving them.
  – MP2. Reason abstractly and quantitatively.
  – MP3. Construct viable arguments and critique the reasoning of others.
  – MP5. Use appropriate tools strategically.
  – MP6. Attend to precision
  – MP7. Look for and make use of structure.
  – MP8. Look for and express regularity in repeated reasoning.
PRR: Standards for Mathematical Practice

• Read your assigned Mathematical Practice.
  – Highlight 3-5 key ideas that strike you as critical to understanding this Mathematical Practice.

• After you have read the practice, jot down:
  – Student behaviors that show engagement with this practice.
  – Teacher actions that will foster the identified student behaviors.
Student Behaviors and Teacher Actions

1. Share your ideas and come to consensus about key ideas that are critical to understanding the standard.

2. Identify 3 student behaviors and 3 related teacher actions as each practice is developed in the math classroom. Create a two-column chart.

3. Include a classroom activity that will target this practice.

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<th>Student Behaviors</th>
<th>Teacher Actions</th>
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MP4: Model with mathematics (Key Ideas of the Standard with Example)
Sharing Summaries of Math Practices

As you share out make sure to highlight...

– The definition of the standard (include examples of key ideas).
– Student behaviors connected to the standard.
– Related teacher actions that would develop this expertise in students.
Keep in mind...

The Mathematical Practice Standards:

• ensure students learn mathematics with deep conceptual understanding.

• are interdependent and do not develop in isolation.

• are not skill-based content that students learn through direct teaching.

• develop over time and emerge through experiences and opportunities provided in the math classroom.
PROFESSIONAL READING & REFLECTION
PRR: The word “Understand or Understanding” appears over 300 times in the CCSSM!

Read page 4-5 in CCSSM starting with Understanding Mathematics.

• Consider the conversations we had today about developing algebraic thinking and about the CCSSM expectations of the Standards for Mathematics Practice.

• Reflect on the message the authors of the CCSSM are sending about their vision for students as well as their vision for mathematics instruction.
Reflection Summary

Summarize some key points and classroom ideas related to the topics or focus standards in this session.

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<th>Focus Topics or Standards</th>
<th>Summary of Key Points</th>
<th>Classroom Ideas to Try</th>
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EARLY COUNTING
Learning Intentions and Success Criteria

We are learning to …

• understand the relationship between numbers and quantities.
• connect counting to cardinality.

We will be successful when

• we can clearly explain the mathematical content in K.CC.4a, K.CC.4b, K.CC.4c, and K.CC.5 and be able to provide examples of the mathematics.
# K-8 Domains & HS Conceptual Categories

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Let’s Watch Some Counting!

3 year old Callum
https://www.youtube.com/watch?v=7r1ZuFuLjlE &feature=youtu.be

• Grab a set of counters and with a neighbor recreate Callum’s counting.
• Where is he accurate?
• Where is he struggling?
Turn and Talk

• How did you and your partner talk about and discuss Callum’s counting skills and abilities?

• Share your thoughts with the other team at your table.
Counting is more than 1,2,3.
Observing Children As They Count

Read your assigned sections.

Be prepared to:
• Summarize each of your sections for your group.
• Provide an example
Milestones to Early Counting

Rote Counting
• Number word list is accurately recited.
• [https://www.youtube.com/watch?v=scu9zzC5U3g](https://www.youtube.com/watch?v=scu9zzC5U3g)

One-to-one Correspondence
• One object paired with another object
• One object paired with one and only one count word

Cardinality
• The last number word stated tells how many there are in the counted set.
Watching Kindergartners Count:

- Josh
- Jason
- Amanda

- Stable & Correct
- Unstable & Correct
- Stable & Incorrect
- Unstable

______________________

Confident

Intentional Viewing
One touch; one count

Correct Counting: one count per object

Incorrect Counting Words

Pointing moves faster than words

Words faster than pointing
Turn and talk:

What are some characteristics of a proficient counter?

Turn to your neighbor and verbalize the progression of object counting skills in young children.
We are learning to ...

- understand the relationship between numbers and quantities.
- connect counting to cardinality.
Disclaimer

Strong Start Math Project
University of Wisconsin-Milwaukee, 2015-2018

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