Strong Start Math Project

Session 3 Handouts
Wednesday June 22, 2016
Racing Bears

Materials

Racing Bears Game board
dot cube (dice)
four teddy bear counters (or another type of counter)
4 counters (any objects-pennies, chips, etc.)

Object

Play is over when players together collect 10 counters

To Play

1. Place a teddy bear at the beginning of each of the four tracks and a counter in the circle at the end of each track
2. Take turns rolling the dot cube and moving any of the bears that number of spaces.
3. The object is to work together to get a bear to the tenth space on any track. When a bear lands exactly on the tenth space, the players take the counter off.
4. Continue playing until all counters have been collected.

The game can be played individually, a group of two, or with the whole family.
Bear Tracks is a board game that provides very young children with opportunities to think about number, in particular cardinality and one-to-one correspondence.

**Directions for Playing**

Children play in pairs. Each player chooses a track on the game board (Appendix A). Alternately the players roll the foam number cube and take a matching number of plastic teddy bear counters or connecting cubes, placing them on their track. Play is collaborative with players helping each other. Play continues until both tracks are filled with counters or cubes.

**Noting the Mathematical Landscape: Openings and Possibilities**

Young children who are learning to count often confuse the number they end on as the name of the last object, rather than the quantity in the set. Rather than moving a counter along a board as in most board games, Bear Tracks is designed to encourage children to examine number as a quantity—cardinal number. The quantity that is rolled is the amount placed on the board. Because the teddy bears (and connecting cubes) come in several colors, small sets appear within the larger ones. Noticing these sets, children may comment after taking five bears, “I have three yellow and two green.” This noticing of sets within sets is to be encouraged. Also note how children determine how many pieces to take after they roll the number cube. Do they count the dots and know that the number they end on is the total quantity of dots? Do they double-tag or skip some dots? Or do they need to place a teddy bear on each dot to determine how many (this is why the game is played with a large foam cube).

**Materials Needed**

- Large foam number cubes—one per pair of children
- Bear Tracks game board (Appendix A)—one per pair of children
- Plastic teddy bears (or other counters such as cubes)
Karen (the teacher): Inez, you roll the cube first. And remember we read the top to tell us how many bears to put on the track.

Inez: It's 1, 2, 3, 4, 5, 6.

Karen: So how many teddy bears do you need to take?

Inez: (Places one teddy bear on each dot.) This many.

Karen: How many is that?

Inez: It's 1, 2, 3, 4, 5, 6.

Karen: OK, it's 6 teddy bears. Let's put them on your track. (Inez places them on the track, one on each square.) You took red and blue teddy bears. How many red ones do you have?

Inez: I have 3.

Karen: And how many blue ones?

Inez: Also 3.

Karen: So you have 3 red ones and 3 blue ones and 6 altogether. OK, it's your turn, Shareema.

Karen's Notes:

Karen asks how many after Inez counts to see if she realizes that the result of her counting is the number contained in the set represented by "6." She does not. In fact, she counts again even after she places the bears one-to-one on the dots.

Karen asks about smaller quantities (the red and blue) to see if Inez subitizes smaller amounts. She does not need to count three.

By summarizing, Karen provides a focus for reflection.
Appendix A

Bear Tracks game board

Make two copies and glue together with paws showing only at the top and bottom of tracks.
Teaching Math in the Primary Grades

The Learning Trajectories Approach

Julie Sarama and Douglas H. Clements

Two kindergarten teachers sit down for lunch during a professional development workshop. One says, “I think it’s ridiculous. The children are still babies. They’re trying to teach them too much.” Her friend nods. Soon they are joined by a colleague from another school, who bubbles, “Isn’t this great? The children are going to know so much more!”

Most of us can sympathize with both perspectives. What should we be teaching in the early grades? Three research findings provide some guidance in mathematics instruction.

1. Learning substantial math is critical for primary grade children.

The early years are especially important for math development. Children’s knowledge of math in these years predicts their math achievement for later years—and throughout their school career. Furthermore, what they know in math predicts their later reading achievement as well (Duncan et al. in press). Given that early math learning predicts later math and reading achievement, math appears to be a core component of learning and thinking.

2. All children have the potential to learn challenging and interesting math.

Primary grade children have an often surprising ability to do abstract math—that is, math that is done by reasoning mentally, without the need for concrete objects. Listen to the worries of this first-grader.

“I find it easier not to do it [simple addition] with my fingers because sometimes I get into a big muddle with them [and] I find it much harder to add up because I am not concentrating on the sum. I am concentrating on getting my fingers right . . . It can take longer to work out the sum [with fingers] than it does to work out the sum in my head.” [In her head, Emily imagined dot arrays. Why didn’t she just use those?] “If we don’t use our fingers, the teacher is going to think, ‘Why aren’t they using their fingers . . . they are just sitting there thinking’ . . . We are meant to be using our fingers because it is easier . . . which it is not.” (Gray & Pitta 1997, 35)

Should the teacher encourage Emily to use concrete objects to solve math problems? Or should she encourage children like Emily to use arithmetic reasoning?

Primary grade children often know, and can definitely learn, far more challenging and interesting math than they are taught in most U.S. classrooms. That does not necessarily mean math pushed down from higher grades. It means letting children invent their own strategies for solving a variety of types of problems. How can teachers best support creative thinking in mathematics?

3. Understanding children’s mathematical development helps teachers be knowledgeable and effective in teaching math.

Children’s thinking follows natural developmental paths in learning math. When teachers understand these paths and offer activities based on children’s progress along them, they build math learning environments that are developmentally appropriate and particularly effective. A useful tool in understanding and supporting the development of children’s mathematical reasoning is a math learning trajectory. There are learning trajectories for mathematics at all age levels, from birth throughout the school years.
and for learning all kinds of content—from specific math concepts such as number and operations to specific science concepts like understanding electricity.

Learning trajectories

Math learning trajectories have three parts: a mathematical goal, a developmental path along which children’s math knowledge grows to reach that goal, and a set of instructional tasks, or activities, for each level of children’s understanding along that path to help them become proficient in that level before moving on to the next level. Let’s examine each of these three parts.

**Goal.** The first part of a learning trajectory is the goal. Goals should include the big ideas of math, such as “numbers can be used to tell us how many, describe order, and measure” and “geometry can be used to understand and to represent the objects, directions, and locations in our world, and the relationship between them” (Clements, Sarama, & DiBiase 2004). In this article, we look at the goal of knowing how to solve a variety of addition and subtraction problems.

**Developmental path.** The second part of a learning trajectory consists of levels of thinking, each more sophisticated than the last, leading to achieving the mathematical goal. That is, the developmental path describes a typical learning route children follow in developing understanding of and skill in a particular mathematics topic.

Learning trajectories are important because young children’s ideas and their interpretations of situations are different from those of adults. Teachers must interpret what the child is doing and thinking and attempt to see the situation from the child’s viewpoint. Knowledge of developmental paths enhances teachers’ understanding of children’s thinking, helping teachers assess children’s level of understanding and offer instructional activities at that level. Similarly, effective teachers consider the instructional tasks from the child’s perspective.

**Instructional tasks.** The third part of a learning trajectory consists of sets of instructional tasks or activities matched to each level of thinking in a developmental progression.

The tasks are designed to help children learn the ideas and practice the skills needed to master that level. Teachers use instructional tasks to promote children’s growth from one level to the next.

Teaching challenging and interesting math

The three research findings—the importance of math learning in the primary grades, all children’s potential to learn math, and teachers’ need to understand children’s learning development—have implications for teaching primary grade math well. We suggest the following approach:

- Know and use learning trajectories.
- Include a wide variety of instructional activities. The learning trajectories provide a guide as to which activities are likely to challenge children to invent new strategies and build new knowledge.
- Use a combination of teaching strategies. One effective approach is to (a) discuss a problem with a group, (b) follow up by having children work in pairs, and then (c) have the children share solution strategies back with the group. Discuss strategies with children in pairs and individually. Differentiate instruction by giving groups or individual children different problem types.

Alexander and Entwistle state that “the early grades may be precisely the time that schools have their strongest effects” (1988, 114). Math is so important to children’s success in school, in the primary grades and in future learning, that it is critical to give children motivating, substantive educational experiences. Learning trajectories are a powerful tool to engage all children in creating and understanding math.

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The National Association of Early Childhood Specialists in State Departments of Education (NAECS/SDE) works to improve instruction, curriculum, and administration in education programs for young children and their families. Of Primary Interest is written by members of NAECS/SDE for kindergarten and primary teachers. The column appears in March, July, and November issues of Young Children and Beyond the Journal (online at www.journal.naeyc.org/btj).

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References


### Learning Trajectory for Addition and Subtraction: Sample Levels of the Developmental Path and Examples of Instructional Tasks

This chart gives simple labels and a sampling of levels in the developmental learning progressions for ages 5 through 7 years. The ages in the first column are not exact indications—children in challenging educational environments often create strategies that are surprisingly sophisticated. The second column describes four main levels of thinking in the addition and subtraction learning trajectory. These levels are samples—there are many levels in between them (for full learning trajectories, see Clements & Sarama 2009 and Sarama & Clements 2009). The third column briefly describes examples of instructional tasks.

<table>
<thead>
<tr>
<th>Age</th>
<th>Developmental path—Sample levels</th>
<th>Instructional tasks</th>
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</thead>
<tbody>
<tr>
<td>5</td>
<td>Find Change. Children find the missing addend ((5 + _ = 7)) by adding on objects.</td>
<td><strong>Word Problems.</strong> For example, say to the children, “You have 5 balls and then get some more. Now you have 7 balls in all. How many more balls did you get?” Children use balls in 2 colors to solve such problems.</td>
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<td>5½</td>
<td>Counting Strategies. Children find sums for joining problems (“You have 8 apples and get 3 more . . .”) and part-part-whole problems (“6 girls and 5 boys . . .”) with finger patterns [counting using fingers and quickly recognizing the quantity] and/or by counting on. <strong>How Many Now? Problems.</strong> For example, have the children count objects as you place them in a box. Ask, “How many are in the box now?” Add 1, repeating the question, then check the children’s responses by counting all the objects. Repeat, checking occasionally. When children are ready, sometimes add 2, and eventually more, objects.</td>
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<td></td>
<td><strong>COUNTING ON.</strong> The teacher asks, “How much is 4 and 3 more?” A child replies, “4 . . . 5, 6, 7 [uses a rhythmic or finger pattern to keep track]. 7!” <strong>COUNTING UP.</strong> A child may solve a missing addend ((3 + _ = 7)) or compare problems by counting up; for example, the child counts “4, 5, 6, 7” while putting up fingers, and then counts or recognizes the 4 fingers raised. Or the teacher asks, “You have 6 balls. How many more do you need to have 8 balls?” The child says, “6, 7 [puts up a finger], 8 [puts up a second finger]. 2!” <strong>Double Compare.</strong> Children compare sums of 2 cards to determine which sum is greater. Encourage the children to use more sophisticated strategies, such as counting on.</td>
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<td>6</td>
<td>Part-Whole. The child has an initial part-whole understanding and can solve all the preceding problem types using flexible strategies (may use some known combinations, such as “5 + 5 is 10”). <strong>Bright Idea.</strong> Using a numeral and a frame with dots, children count on from the numeral to identify the total amount. They then move forward a corresponding number of spaces on a game board. <strong>Hidden Objects.</strong> Hide 4 counters under a dark cloth and show children 7 counters. Tell them that 4 counters are hidden and challenge them to tell you how many counters there are in all. Or tell the children there are 11 counters in all, and ask how many are hidden. Have the children discuss their solution strategies. Repeat with different sums. <strong>Barkley’s Bones.</strong> Children determine the missing addend in problems such as (4 + _ = 7). <strong>Twenty-one.</strong> Play this card game, whose object is to have the sum of one’s cards be 21 or as close as possible without exceeding 21. An ace is worth either 1 or 11, and cards for 2 through 10 are worth their face value. A child deals everyone 2 cards, including herself.</td>
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<tr>
<td>7</td>
<td>Deriver. The child uses flexible strategies and derived combinations (“(7 + 7 = 14), so (7 + 8 = 15)”) to solve all types of problems. <strong>Multidigit Addition and Subtraction.</strong> “What’s (28 + 35)?”</td>
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Learning Trajectory
Developmental Levels for “Counting”

The ability to count with confidence develops over the course of several years. Beginning in infancy, children show signs of understanding number. With instruction and number experience, most children can count fluently by age 8, with much progress in counting occurring in kindergarten and first grade. Most children follow a natural developmental progression in learning to count with recognizable stages or levels. This developmental path can be described as part of a learning trajectory.

<table>
<thead>
<tr>
<th>Level</th>
<th>Level Name</th>
<th>Description</th>
<th>Notes</th>
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<tbody>
<tr>
<td>1</td>
<td>Pre-Counter</td>
<td>A child names some number words in no apparent order and without meaning.</td>
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<td>2</td>
<td>Chanter</td>
<td>A child sing-songs numbers often in some order, but it is a song and without meaning of quantity or counting.</td>
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<td>3</td>
<td>Reciter</td>
<td>A child verbally recites number names as separate words with the intention to count, but does not necessarily recite the correct order.</td>
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<td>4</td>
<td>Reciter (10)</td>
<td>A child verbally counts to 10 with some correspondence with objects. The child may point to objects to count a few items but then often loses track.</td>
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<td>5</td>
<td>Corresponder</td>
<td>A child can keep one-to-one correspondence between counting words and objects—at least for small groups of objects laid in a line. When asked “how many,” the child often recounts the objects starting over with one each time.</td>
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<td>6</td>
<td>Counter (Small Numbers)</td>
<td>A child begins to count meaningfully. The child accurately counts a given set of objects to 5 and answers the “how many” question with the last number counted without needing to recount the objects.</td>
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<tr>
<td>7</td>
<td>Producer—Counter To (Small Numbers)</td>
<td>When asked to show a specific number of objects, a child can accurately produce or make a set of objects up to 5.</td>
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<td>8</td>
<td>Counter (10)</td>
<td>A child accurately counts structured arrangements of objects to 10. He or she may be able to draw representations for quantities up to 10. The child can also find the number just after or just before another number, but only by counting up from 1.</td>
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<tr>
<td>9</td>
<td>Counter and Producer Counter to (10+)</td>
<td>Child accurately counts and produces sets to 10 and beyond to 30, keeping track of objects that have and have not been counted. Child draws representations to 10, then to 20 and 30, and can find the next number to 20 or 30. Child recognizes errors in others’ counting and can eliminate most errors in one’s own counting.</td>
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<td></td>
<td>Description</td>
<td>Example</td>
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<td>-----------------------------------------------------------------------------</td>
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<tr>
<td>10</td>
<td>Counter Backward from 1</td>
<td>The child is able to count backwards from 10.</td>
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<td>11</td>
<td>Counter from N (N+1, N-1)</td>
<td>The child begins to count on from numbers other than one, either in verbal counts or with objects. The child can determine the number just before or just after another number quickly without having to start counting back at one.</td>
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<td>12</td>
<td>Skip-Counting by 10s to 100</td>
<td>The child can count by tens to 100.</td>
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<tr>
<td>13</td>
<td>Counter to 100</td>
<td>The child can count by ones through 100, including knowing the decade transitions from 39 to 40, 49 to 50, and so on, starting at any number.</td>
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<tr>
<td>14</td>
<td>Counter on using patterns</td>
<td>The child keeps track of counting acts by using numerical patterns or movements, such as tapping as he or she counts.</td>
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<tr>
<td>15</td>
<td>Skip Counter</td>
<td>The child can count by five and twos with understanding.</td>
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<td>16</td>
<td>Counter of Imagined Items</td>
<td>The child can count mental images of hidden objects.</td>
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<td>17</td>
<td>Counter On Keeping Track</td>
<td>The child can keep track of counting acts numerically with the ability to count on (one to four counts) from a given number.</td>
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<tr>
<td>18</td>
<td>Counter of Quantitative Units</td>
<td>The child can count unusual units such as &quot;wholes&quot; when shown combinations of wholes and parts. For example when shown three whole plastic eggs and four halves, a child at this level will say there are five whole eggs.</td>
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<tr>
<td>19</td>
<td>Counter to 200</td>
<td>The child counts accurately to 200 and beyond, recognizing the patterns of ones, tens, and hundreds.</td>
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<tr>
<td>20</td>
<td>Number Conserver</td>
<td>The child demonstrates the ability to conserve number. She or he understands that a number is unchanged even if a group of objects is rearranged. For example, if there is a row of ten buttons, the child understands there are still ten without recounting, even if they are rearranged in a long row or a circle.</td>
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