Strong Start Math Project

Session 6 Handouts
Monday June 27, 2016
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<th>Mathematics Teaching Practices</th>
<th>Highlights of Key Points</th>
<th>Narrative Case Examples (line numbers &amp; summarize)</th>
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<td>Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.</td>
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<td>Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.</td>
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<td>Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.</td>
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<td>Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.</td>
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<td>Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students’ reasoning and sense making about important mathematical ideas and relationships.</td>
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<td>Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.</td>
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<td>Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.</td>
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<td>Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.</td>
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Exploring Addition
The Case of Ms. Bouchard and the Hungry Caterpillar Task

It was early in the school year as the first grade class sat on the rug and listened to Ms. Bouchard read “The Very Hungry Caterpillar” by Eric Carle. Ms. Bouchard used the story as a context for doing mathematics with her young learners. In using the children’s literature book, she hoped her students might see mathematics as a way to answer questions that emerge in their everyday activities. After discussing the story, she commented, “I wonder how many pieces of fruit were eaten through by that caterpillar?” This question prompted a task, shown below, for engaging her students in mathematical reasoning and problem solving. It was well aligned with her learning goals for the students to see and model the mathematical elements in a contextual problem, to understand addition as putting together sets of objects, and to understand that counting can be used to answer “how many” questions when combining two or more sets of objects.

On Monday, the hungry caterpillar ate through one apple, but he was still hungry. On Tuesday he ate through two pears, but he was still hungry. On Wednesday he ate through three plums. On Thursday he ate through four strawberries. On Friday he ate through five oranges.

How many pieces of fruit did the hungry caterpillar eat during the week?

Before allowing the students to work on the problem, Ms. Bouchard asked a student to retell the part of the story about the fruit to the whole class. Then pairs of students retold it to each other. Next they discussed what the question was asking and what their answer would tell them. If the students wanted, they could use cubes or other math tools to help them work on the task. They also had access to blank paper, crayons, and markers as Ms. Bouchard expected all of the students to make a picture that represented their thinking and their answer.

As the students worked, Ms. Bouchard made her way around the classroom asking students questions and making note of their approaches so she could decide which students she wanted to present their work, and in which order, later during the whole class discussion. She looked for evidence that the picture showed both the story and how the student found the answer. She also noted which students wrote equations on their paper.

While some students immediately started to draw pictures on their paper, a few students struggled to organize the information on their papers. Ms. Bouchard would ask these students, “So, tell me about your picture” or “How does the picture show your thinking?” or “How does the picture show the story?” Other students, like Mikaela, used cubes to figure out the answer, but struggled with what to put on their paper. When the teacher asked her how she got her answer, Mikaela replied, “I know the answer is 15 but I don’t know what’s in my head.” Through further questioning, the student explained that she had put out the cubes and then counted them all up. Ms. Bouchard encouraged her to draw a picture of what she had just described. Later when the teacher glanced at Mikaela’s paper it contained 15 tally marks and the number sentence \(5 + 5 + 5 = 15\), which showed the answer but did not show her thinking nor how the numbers related to the story.

Before holding a whole class discussion, Ms. Bouchard asked students to describe their picture to a partner. This gave all students a chance to articulate and share their own thinking. It also prompted students to begin orienting themselves to the thinking of other students, certainly not an easy task for first grade students but an important step in moving the class to more meaningful discourse and shared understanding of mathematical ideas.

Irene was asked to present her work first since it clearly modeled the context of the story by showing the number of each type of fruit with both a numeral and a picture (see student work at the bottom of the next page). Ms. Bouchard put her work on the document camera and then asked her to explain her picture and how she figured out the total. Irene demonstrated how she counted all the fruit by ones to get her answer of 15. Many students had a picture like Irene’s. Maya presented next as it showed another aspect of the story, the days of the week, and moved beyond pictures to using squares as a generalized symbol to represent the fruit. After the first two presentations, the teacher engaged the whole class in a comparison of the two approaches.

Written by DeAnn Huinker (University of Wisconsin-Milwaukee) and Beth Schefelker (School District of South Milwaukee), based on lessons taught by Meribeth Bouchard and Kari Derr (School District of South Milwaukee). This case is intended to support the Guiding Principle on Teaching and Learning in Principles to Actions: Ensuring Mathematical Success for All (National Council of Teachers of Mathematics, 2014).
Teacher: What is similar and different about Irene’s picture and Maya’s picture?
Marcus: Well, Irene drew pictures of the fruit but Maya didn’t, she drew squares.
Teacher: Is that okay, to draw squares?
Marcus: Yeah, because you know it’s suppose to be the fruit.
Irene: I like the pictures better, because you can see what kind of fruit it was.
Teacher: What else do you notice?
Gabe: Maya wrote out the days like our calendar.
Teacher: So both pictures tell us something about the story. Irene’s picture shows the different types of fruit and how many pieces of each and Maya’s picture shows how many pieces of fruit where eaten on the different days but it doesn’t show what kind of fruit was eaten.

Aiden presented next as his approach showed the context and he also wrote a number sentence. This gave the class the opportunity to discuss how this task was related to the operation of addition and how counting all the objects to find the total was the same as adding $1 + 2 + 3 + 4 + 5$. Then Cole was asked to share his work; he had written $10 + 5 = 15$ as his number sentence. Here is an excerpt from the discussion of his work.

Teacher: I see you wrote $10 + 5 = 15$. Why didn’t you write a number sentence like Aiden?
Cole: I didn’t know you could write a number sentence with all those plus signs.
Teacher: What do others think of Cole’s comment?
Sophia: I think it’s okay because we’re putting all of the numbers together.
Teacher: Let’s look at Cole’s number sentence. Cole, can you explain how your number sentence matches your picture? I’d like everyone to listen carefully and I’ll call on someone to repeat what he says.
Cole: I knew that the 2 pears and 3 plums were 5 so I circled them, that’s five. Then I added up the strawberries, oranges, and the apple and that was 10. So I added $10 + 5$.
Teacher: So, the arrow shows your thinking of how you put the apple with the strawberries and oranges.
Let’s see. Who thinks they can explain to us why Cole added ten plus five? Hannah?
Hannah: Cole put the pears and plums together and that was five pieces of fruit. Then Cole put together the apple, the strawberries, and oranges and that was ten pieces of fruit. Then he added it up.

Toward the end of the lesson, Ms. Bouchard had Evan show his work. She asked the students to turn and talk about it with a partner and discuss why they thought Evan wrote $6 + 9 = 15$ and then they talked about it as a class. To close the lesson, Ms. Bouchard asked each student to look at their picture and to revise it so that it would better show their thinking using ideas they got by looking at and talking about the work of their classmates.
Mathematics Teaching Practice #3

Use and Connect Mathematical Representations

Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

Representational Actions

**Physical**
Use concrete objects to show, study, act upon, or manipulate mathematical ideas (e.g., cubes, counters, tiles, paper strips).

**Visual**
Illustrate, show, or work with mathematical ideas using diagrams, pictures, number lines, graphs, and other math drawings.

**Contextual**
Situate mathematical ideas in everyday, real-world, imaginary, or mathematical situations and contexts.

**Symbolic**
Record or work with mathematical ideas using numerals, variables, tables, and other symbols.

**Verbal**
Use language (words and phrases) to interpret, discuss, define, or describe mathematical ideas, bridging informal and formal mathematical language.


http://www.nctm.org/pta

Developed by DeAnn Huinker (07.06.2015).
Represent It!

- Draw It
- Write it with numbers
- Build It
- Write a story
- Talk It

Developed by Beth Schefelker (South Milwaukee School District) and DeAnn Huinker (University of Wisconsin-Milwaukee)