Agenda

• Story Problem Structures
• K.OA.1, and Part 2 of K.OA.2, 1.OA.1, and 2.OA.1
  – Levels of Thinking
  – Written Representations
• Lunch
• Mathematical Curiosity
Story Problem Practice

- Stand up – Find a grade level partner!
- Brainstorm contexts for story problems with a grade level partner.
- With a your table partner, write a word problem for each problem type.

Keep your problem above the dotted line.
DEVELOPMENTAL LEVELS OF REASONING
Part 2 of these standards

- **K.OA.1**: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.
- **K.OA.2**: ...add and subtract within 10, e.g., by using objects or drawings to represent the problem.
- **1.OA.1**: ...by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
- **2.OA.1**: ...by using drawing and equations with a symbol for the unknown number to represent the problem.

How are students expected to use objects and drawings to solve and represent these problem subtypes according to the CCSSM?
How might a Kindergarten child solve this problem?

There were 6 cookies on the plate. Wonza put 5 more cookies on the plate. How many cookies are on the plate now?

Grab some counters and model a kindergartner’s thinking.
What are the levels of reasoning?

• Read & Highlight your section of “Developmental Levels of Reasoning” on pp. 37-38 of OA Progressions

• Share in teams of 3.
  – Share 2-3 ideas you highlighted and why you highlighted them.
  – Using your whiteboard, share an example of student reasoning at your developmental level.

• Share your level in the following order:
  – Direct Modeling ➔ Counting On/Counting Down ➔ Numerical Reasoning
Developmental Levels

• Level 1 Direct Modeling
  – Child models the situation or action step by step using objects, pictures, and uses a count-count-count approach.

• Level 2 Counting On
  – Child visualizes the quantities and then uses a counting-on approach; often uses fingers to keep track of the counts.

• Level 3 Convert to An Easier Equivalent Problem-Numerical Reasoning
  – Child uses number relationships to strategically work with quantities; such as decomposing numbers and finding easier ways to recompose them.
LEVEL 1 THINKING
Level 1 Strategies

• Joining All
• Joining To
• Separating From
• Separating To
• Matching
• Trial and Error
Practicing Level 1 Thinking

With a partner, a device (phone, tablet, etc.), counters, the handout, and your CGI book:

• Read the portion of the chapter about each of the strategies for Level 1 thinking.

• If applicable, use your device to watch a video clip of a student.

• Model how a child would think through the corresponding problem using that strategy on the handout.

• Flip to page 22 to check your modeling.

• Repeat with the next strategy for Level 1.
What observations are you making about Level 1 strategies?

• Takes more than simply using manipulatives.
• Must **explicitly** model the action or the relationships in the problem.
### Morning Reflection/Summary

- Summarize some key points and classroom ideas related to the topics or focus standards in this session.

<table>
<thead>
<tr>
<th>Focus Topics or Standards</th>
<th>Summary of Key Points</th>
<th>Classroom Ideas to Try</th>
</tr>
</thead>
<tbody>
<tr>
<td>Levels of Thinking Level 1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
MATHEMATICAL CURIOSITY: MAZES AND PASCAL’S TRIANGLE
Magic maze

The maze in the left has only one entrance and seven exits. Mr. Johnson wants to put his ice cream cart near the exit where most visitors come out, so that he could sell more ice cream.

The following rules govern the maze: at the entrance each visitor gets a die. At every crossroad in the maze the visitor rolls the die. If the number on top is even, the visitor goes left. If it is odd, the visitor goes right.

Where do you think Mr. Johnson should place his ice cream cart?
Pascal’s triangle

Blaise Pascal (1623 – 1662)

*Traité du triangle arithmétique* (1653)

Find a pattern, and try to explain it!
LEVEL 2 STRATEGIES
Children’s Mathematics, p. 34-36

In your notebook, discuss connections you are making between Level 1 thinking and the story problem types.
Level 2 Strategies

- Counting on from the first number
- Counting on from the larger number
- Counting on to
- Counting down
- Counting down to
Practicing Level 2 Thinking

With a partner, a device (phone, tablet, etc.), counters, the handout, and your CGI book:

- Read the portion of the chapter about each of the strategies for Level 1 thinking.
- If applicable, use your device to watch a video clip of a student.
- Model how a child would think through the corresponding problem using that strategy on the handout.
- Flip to page 27 to check your modeling.
- Repeat with the next strategy for Level 1.
What observations are you making about Level 2 strategies?

- Indicate an ability to connect to numbers as more abstract quantities. They realize it is not necessary to build each quantity in a given situation.
- Often involve double counting of some kind – counting and keeping track of the count at the same time.
- When/If a student uses objects, they are often used to keep track of the count instead of representing a quantity in the problem.

**Even after children become very efficient with Level 2 strategies and thinking, they may occasionally fall back to Direct Modeling/Level 1 Thinking.**
Looking at Student Work

Allison and her brother frosted all of the cookies that their mom made for the party. Allison decorated 20 cookies. Her brother decorated 3 cookies. How many cookies will there be at the party?

Which piece of student work matches each Level? Explain to each other using the language from the reading.

p. 44-45 of Children’s Mathematics.
Read: *Children’s Mathematics: Cognitively Guided Instruction* p.29: Distinction Between Counting and Modeling Strategies

• What are key understandings children bring from K and 1st Grade that will support a successful transition to Level 2 strategies?

• What shifts to instruction are we being asked to make to facilitate Level 2 thinking?
Learning Intention & Success Criteria

We are learning

• the CCSSM expectations around story problem structures.
• that students progress through levels of reasoning as they solve story problems.
• how we might support students’ representations of their thinking.
Afternoon Reflection/Summary

- Summarize some key points and classroom ideas related to the topics or focus standards in this session.

<table>
<thead>
<tr>
<th>Focus Topics or Standards</th>
<th>Summary of Key Points</th>
<th>Classroom Ideas to Try</th>
</tr>
</thead>
<tbody>
<tr>
<td>Levels of Thinking Level 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Disclaimer

**Strong Start Math Project**
University of Wisconsin-Milwaukee, 2015-2018

This material was developed for the *Strong Start Math* project through the University of Wisconsin-Milwaukee, Center for Mathematics and Science Education Research (CMSER). This material may be used by schools to support learning of teachers and staff provided appropriate attribution and acknowledgement of its source. Other use of this work without prior written permission is prohibited—including reproduction, modification, distribution, or re-publication and use by non-profit organizations and commercial vendors.

This project was supported through a grant from the Wisconsin ESEA Title II, Part B, Mathematics and Science Partnerships.